Grain screening in semi-bounded plasmas

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Abstract. The screening potentials of single grain embedded into semi-bounded plasmas are calculated. Limiting cases of collisionless plasma as well as plasma described in the drift-diffusion approach are considered. It is assumed that plasma boundary does not absorb reflecting particles. We treat the grain as point-like particle with appropriate singular sink in the equations describing plasma dynamics that makes it possible to take into account plasma particles absorption by grain. Such model approach allows recovering the results for screening potentials calculated with regard to finite size of grain within the linear approximation in the case of unbounded plasmas and can be easily extended to the case of bounded system. It turns out to be that plasma dynamics can considerably influence the asymptotic behavior of screening potential. Modifications of such potentials due to boundary influence are studied in detail. Particularly, the presence of asymptotic inversely proportional to the third power of the distance parallel to the boundary is shown.

Keywords: dusty plasma, grain screening, semi-bounded plasma
PACS: 52.27 Lw

Let us consider semi-bounded weakly-ionized plasma ($z > 0$) with a stationary grain at the point $\vec{r}_0 = (0,0,z_0)$. We assume that the grain absorbs all encountered electrons and ions. If the ionization and recombination processes can be neglected the continuity equations for plasma particles are the following

$$\nabla\vec{\Gamma}_\sigma = -S\delta(\vec{r}-\vec{r}_0), \tag{1}$$

where

$$\vec{\Gamma}_\sigma = -\frac{e_\sigma n_\sigma}{m_\sigma \nu_\sigma}\nabla\phi - D_\sigma \nabla n_\sigma, \tag{2}$$

Here S is the intensity of the sink describing plasma particle absorption by grain, the rest of notation is traditional.

The effective grain electric potential is governed by Poisson equation

$$\Delta\phi = -4\pi \sum_{\sigma=e,i} e_\sigma n_\sigma - 4\pi q\delta(\vec{r}-\vec{r}_0), \tag{3}$$

where q is the grain charge. The quantities S and q can be determined from the solution of the appropriate nonlinear boundary-value problem (see, for example [1] and references sited therein). In what follows we assume that S and q are known.

Linearizing Eqs. (1,3) one has

$$\frac{e_\sigma n_0}{T_\sigma}\Delta\phi + \Delta\delta n_\sigma = \frac{S}{D_\sigma}\delta(\vec{r}-\vec{r}_0), \tag{4}$$

$$\Delta\phi = -4\pi \sum_{\sigma=e,i} e_\sigma \delta n_\sigma - 4\pi q\delta(\vec{r}-\vec{r}_0), \tag{5}$$

where $\delta n_\sigma(\vec{r}) = n_\sigma(\vec{r}) - n_0$ is the particle density perturbation, $e_e = -e_i = -e$, where e is the elementary charge.

In the external medium ($z < 0$) electric potential satisfi es the equation

$$\Delta\tilde{\phi} = 0, \tag{6}$$

which has to be solved with the following boundary conditions.

$$\tilde{\phi}(\vec{r}_\perp, z=-0) = \phi(\vec{r}_\perp, z=+0), \qquad \tilde{\varepsilon}\frac{\partial\tilde{\phi}(\vec{r})}{\partial z}\bigg|_{z=-0} = \frac{\partial\phi(\vec{r})}{\partial z}\bigg|_{z=+0}, \qquad \vec{\Gamma}_\sigma(\vec{r}_\perp, z=+0) = 0, \tag{7}$$

where $\tilde{\varepsilon}$ is the dielectric constant of the external medium.

Eqs. (4,5) can be solved using the specular continuation of electric potential and density perturbation to the region $z < 0$ [2]. Such continued quantities Φ and $\bar{n}_\sigma$ satisfy the equations

$$\frac{e_\sigma n_0}{T_\sigma}\Delta\Phi + \Delta\delta\bar{n}_\sigma = \frac{S}{D_\sigma}(\delta(\vec{r}-\vec{r}_0)+\delta(\vec{r}-\vec{r}_0^{\,+})), \tag{8}$$

$$\Delta\Phi = -4\pi\sum_{\sigma=e,i} e_\sigma\delta\bar{n}_\sigma - 4\pi q(\delta(\vec{r}-\vec{r}_0)+\delta(\vec{r}-\vec{r}_0^{\,+})), \tag{9}$$

where $\vec{r}_0^{\,+} = (0,0,-z_0)$.

In the $\vec{k}$-representation the solution of Eqs. (8,9) is the following

$$\Phi_k = \frac{4\pi\tilde{q}\left(e^{-ik_z z_0}+e^{ik_z z_0}\right)}{k^2+k_D^2} - \frac{2\tilde{\varepsilon}k_\perp\phi_{k_\perp}(+0)}{k^2+k_D^2} - \frac{4\pi\tilde{S}}{k^2}\left(e^{-ik_z z_0}+e^{ik_z z_0}\right), \tag{10}$$

where

$$\tilde{S} = \frac{eS}{k_D^2}\left(\frac{1}{D_i}-\frac{1}{D_e}\right), \qquad \tilde{q} = q+\tilde{S}, \qquad k_D^2 = k_{Di}^2+k_{De}^2, \qquad k_{D\sigma}^2 = \frac{4\pi e_\sigma^2 n_\sigma}{T_\sigma}. \tag{11}$$

Notice that $\tilde{S}$ can be treated as an effective charge in the unscreened part of the potential. This quantity can be related to the grain charge as $\tilde{S} = \alpha q$, where α has to be determined from the solution of the nonlinear problem. As was shown in [1], $\alpha \ll 1$ in the case of large grain size and $\alpha \leq 0.5$ for small grain sizes.

The inverse Fourier transformation gives

$$\phi(\vec{r}) = \frac{\tilde{q}}{r_-}e^{-k_D r_-} + \frac{\tilde{q}}{r_+}e^{-k_D r_+} - \frac{\tilde{S}}{r_-} - \frac{\tilde{S}}{r_+} - \\ - 2\tilde{\varepsilon}\tilde{q}\int_0^\infty dk_\perp \frac{k_\perp^2 J_0(k_\perp r_\perp)e^{-z_+\sqrt{k_\perp^2+k_D^2}}}{\sqrt{k_\perp^2+k_D^2}(\sqrt{k_\perp^2+k_D^2}+\tilde{\varepsilon}k_\perp)} + 2\tilde{\varepsilon}\tilde{S}\int_0^\infty dk_\perp \frac{k_\perp J_0(k_\perp r_\perp)e^{-z\sqrt{k_\perp^2+k_D^2}-z_0k_\perp}}{\sqrt{k_\perp^2+k_D^2}+\tilde{\varepsilon}k_\perp}, \tag{12}$$

where $J_0(k_\perp r_\perp)$ is the Bessel function of zeroth order, $z_\pm = z \pm z_0$, $r_\pm = \sqrt{r_\perp^2+z_\pm^2}$.

The first and the third terms in Eq. (12) represent the effective potential in unbounded plasma which consists of the Debye and the Coulomb parts with the appropriate effective charge [3], [4]. The second and the forth terms describe the contribution of the charge image. The last two terms (in what follows I_1 and I_2) apparently are related to the charge induced at the boundary surface. In the case $S = 0$ Eq. (12) reduces to the potential of stationary charge in semi-bounded plasma [2].

In the case $r_\perp k_D/\tilde{\varepsilon} \gg 1$

$$I_1 \approx 2\frac{\tilde{q}\tilde{\varepsilon}}{k_D^2}\frac{e^{-z_+k_D}}{r_\perp^3},$$

at $r_\perp k_D/\tilde{\varepsilon} \gg 1$ and $z_0 k_D/\tilde{\varepsilon} \gg 1$

$$I_2 \approx 2\frac{\tilde{S}\tilde{\varepsilon}}{k_D}\frac{z_0 e^{-zk_D}}{(z_0^2+r_\perp^2)^{3/2}}$$

The induced part of the potential in the point $\vec{r}_0$ is given by

$$\phi_{ind}(\vec{r}_0) = -k_D\tilde{q} + \frac{\tilde{q}}{2z_0}e^{-2k_D z_0} - \frac{\tilde{S}}{2z_0} - 2\tilde{\varepsilon}\tilde{q}\int_0^\infty dk_\perp \frac{k_\perp^2 e^{-2z_0\sqrt{k_\perp^2+k_D^2}}}{\sqrt{k_\perp^2+k_D^2}(\sqrt{k_\perp^2+k_D^2}+\tilde{\varepsilon}k_\perp)} + \\ + 2\tilde{\varepsilon}\tilde{S}\int_0^\infty dk_\perp \frac{k_\perp e^{-z_0(\sqrt{k_\perp^2+k_D^2}+k_\perp)}}{\sqrt{k_\perp^2+k_D^2}+\tilde{\varepsilon}k_\perp}. \tag{13}$$

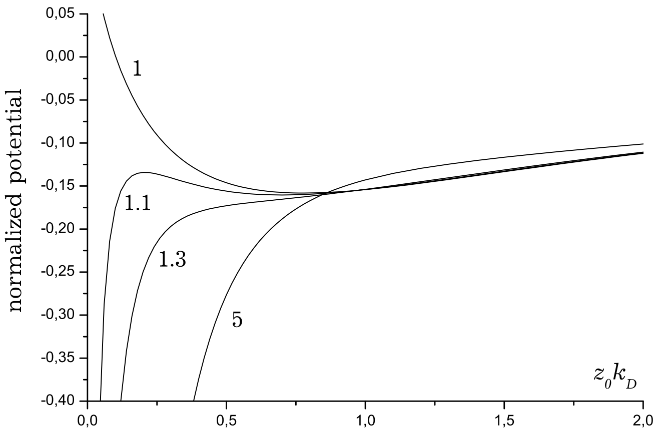

FIGURE 1. Dependence of $\phi_{ind}/qk_D-(1+\alpha)$ on z_0k_D at $\alpha=0.5$ and various values of ε (1; 1,1; 1.3; 5)

Fig. 1 shows typical behaviour of the dimensionless induced potential $\phi_{ind}/qk_D-(1+\alpha)$ on the distance z_0k_D at various values of ε. As follows from the fi gure at $\varepsilon\sim1$ the potential is nonmonotonic that will generate nonmonotonic density profi le for grains near the boundary.

If the plasma under consideration can be described in the collisionless approximation, the stationary Vlasov equation has the form

$$\left(\vec{\upsilon}\frac{\partial}{\partial\vec{r}}-\frac{e_\sigma}{m_\sigma}\nabla\phi(\vec{r})\frac{\partial}{\partial\vec{\upsilon}}\right)f_\sigma(\vec{r},\vec{\upsilon})=-\delta(\vec{r})\sigma_\sigma(\upsilon)\upsilon f_\sigma(\vec{r},\vec{\upsilon}). \tag{14}$$

The singular sink is introduced in the right-hand side of this equation in order to describe plasma particles absorption by grain. In the case under consideration the intensity of sink is determined by the charge cross-section $\sigma_\sigma(q,\upsilon)$ (see, for example [4]). Assuming, that the perturbations introduced by the sink are small it is possible to use the linear approximation, i.e. $f_\sigma(\vec{r},\vec{\upsilon})=f_{0\sigma}(\upsilon)+\delta f_\sigma(\vec{r},\vec{\upsilon})$, where $f_{0\sigma}(\upsilon)$ is the Maxwellian distribution

It is easy to show that the $\vec{k}$-representation of the specularly extended perturbation has the form

$$\delta f_{\sigma\vec{k}}(\vec{\upsilon})=-\frac{e_\sigma}{T_\sigma}\Phi_{\vec{k}}f_{0\sigma}(\upsilon)+i\frac{\sigma_\sigma(\upsilon)\upsilon f_{0\sigma}(\upsilon)}{\vec{k}\vec{\upsilon}-i0}(e^{-ik_zz_0}+e^{ik_zz_0}). \tag{15}$$

that generates the following charge density distribution

$$\delta\rho_{\sigma\vec{k}}=-\frac{e_\sigma^2n_\sigma}{T_\sigma}\Phi_{\vec{k}}-\frac{2\pi^2}{k}e_\sigma n_\sigma\int d\upsilon\upsilon^2\sigma_\sigma(\upsilon)f_{0\sigma}(\upsilon)(e^{-ik_zz_0}+e^{ik_zz_0}). \tag{16}$$

Substituting Eq. (16) into the Poisson equation for the extended potential one has

$$\Phi_{\vec{k}}=\frac{4\pi q}{k^2+k_D^2}(e^{-ik_zz_0}+e^{ik_zz_0})-\frac{8\pi^3(A_i+A_e)}{k(k^2+k_D^2)}(e^{-ik_zz_0}+e^{ik_zz_0})-\frac{2\tilde{\varepsilon}k_\perp\Phi_{k_\perp}(+0)}{k^2+k_D^2}, \tag{17}$$

where

$$A_\sigma=e_\sigma n_\sigma\int\upsilon^2\sigma_\sigma(\upsilon)f_{0\sigma}(\upsilon)d\upsilon.$$

With regard to the boundary conditions (7)

$$\phi(r)=q\frac{e^{-k_Dr_-}}{r_-}+q\frac{e^{-k_Dr_+}}{r_+}-2\pi(A_i+A_e)\big(f(k_Dr_-)+f(k_Dr_+)\big)-$$
$$-2\tilde{\varepsilon}q\int_0^\infty dk_\perp\frac{k_\perp^2J_0(k_\perp r_\perp)e^{-z_+\sqrt{k_\perp^2+k_D^2}}}{\sqrt{k_\perp^2+k_D^2}(\sqrt{k_\perp^2+k_D^2}+\tilde{\varepsilon}k_\perp)}+4\pi\tilde{\varepsilon}(A_i+A_e)\int_0^\infty dk_\perp\frac{k_\perp^2J_0(k_\perp r_\perp)e^{-z\sqrt{k_\perp^2+k_D^2}}}{\sqrt{k_\perp^2+k_D^2}+\tilde{\varepsilon}k_\perp}\int_{-\infty}^{\infty}\frac{dk_z\cos(k_zz_0)}{k(k^2+k_D^2)}, \tag{18}$$

where $f(x) = (e^{-x}Ei(x) - e^{x}Ei(-x))/x$.

As is seen, the structure of the potential (18) is similar to that one in the case of weakly-ionized plasma.

In the case of Maxwellian distribution

$$A_i = e_i n_i \frac{a^2}{4}\left(1 + 2\frac{z}{t}\right), \qquad A_e = e_e n_e \frac{a^2}{4}\left(2e^{-z}\sqrt{\frac{z}{\pi}} + (1-2z)(1 - erf(\sqrt{z}))\right), \tag{19}$$

where $erf(x) = \frac{2}{\sqrt{\pi}}\int\limits_0^x e^{-t^2}dt$ is the error function.

Notice, that in the case of unbounded plasma

$$\phi(r) = \frac{q}{r}e^{-k_D r} - 2\pi(A_i + A_e)f(k_D r), \tag{20}$$

that is in agreement with the result obtained in [5].

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